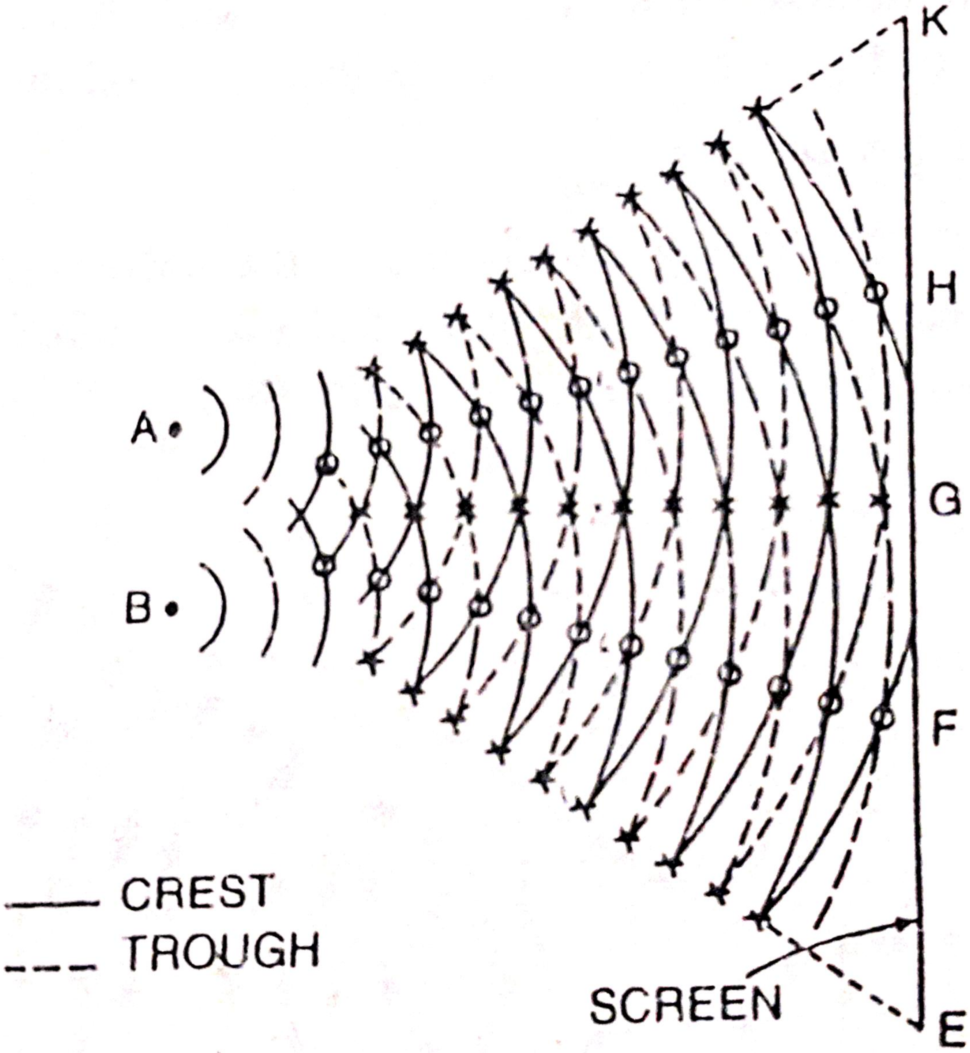
INTERFERENCE

8.1 INTRODUCTION

The phenomenon of interference of light has proved the validity of the wave theory of light. Thomas Young successfully demonstrated his experiment on interference of light in 1802. When two or more wave trains act simultaneously on any particle in a medium, the displacement of the particle at any instant is due to the superposition of all the wave trains. Also, after the superposition, at the region of cross over, the wave trains emerge as if they have not interfered at all. Each wave train retains its individual characteristics. Each wave train behaves as if others are absent. This principle was explained by Huygens in 1678.


K
H
A
G
B
F
CREST
TROUGH
SCREEN
E

The phenomenon of interference of light is due to the superposition of two trains within the region of cross over. Let us consider the waves produced on the surface of water. In Fig. 8.1 points A and B are the two sources which produce waves of equal amplitude and constant phase difference. Waves spread out on the surface of water which are circular in shape. At any instant, the particle will be under the action of the displacement due to both the waves. The points shown by circles in the diagram will have minimum displacement because the crest of one wave falls on the trough of the other and the resultant displacement is zero. The points shown by crosses in the diagram will have maximum displacement because, either the crest of one will combine with the crest of the other or the trough of one will combine with the trough of the other. In such a case, the amplitude of the displacement is twice the amplitude of either of the waves. Therefore, at these points the waves **reinforce** with each other. As the intensity (energy) is directly proportional to the square of the amplitude ($I \propto A^2$) the intensity at these points is four times the intensity due to one wave. It should be remembered that **there is no loss of energy due to interference.** The energy is only transferred from the points of minimum displacement to the points of maximum displacement.

8.2 YOUNG'S EXPERIMENT

In the year 1802, Young demonstrated the experiment on the interference of light. He allowed sunlight to fall on a pinhole S and then at some distance away on two pinholes A and B (Fig. 8.2).

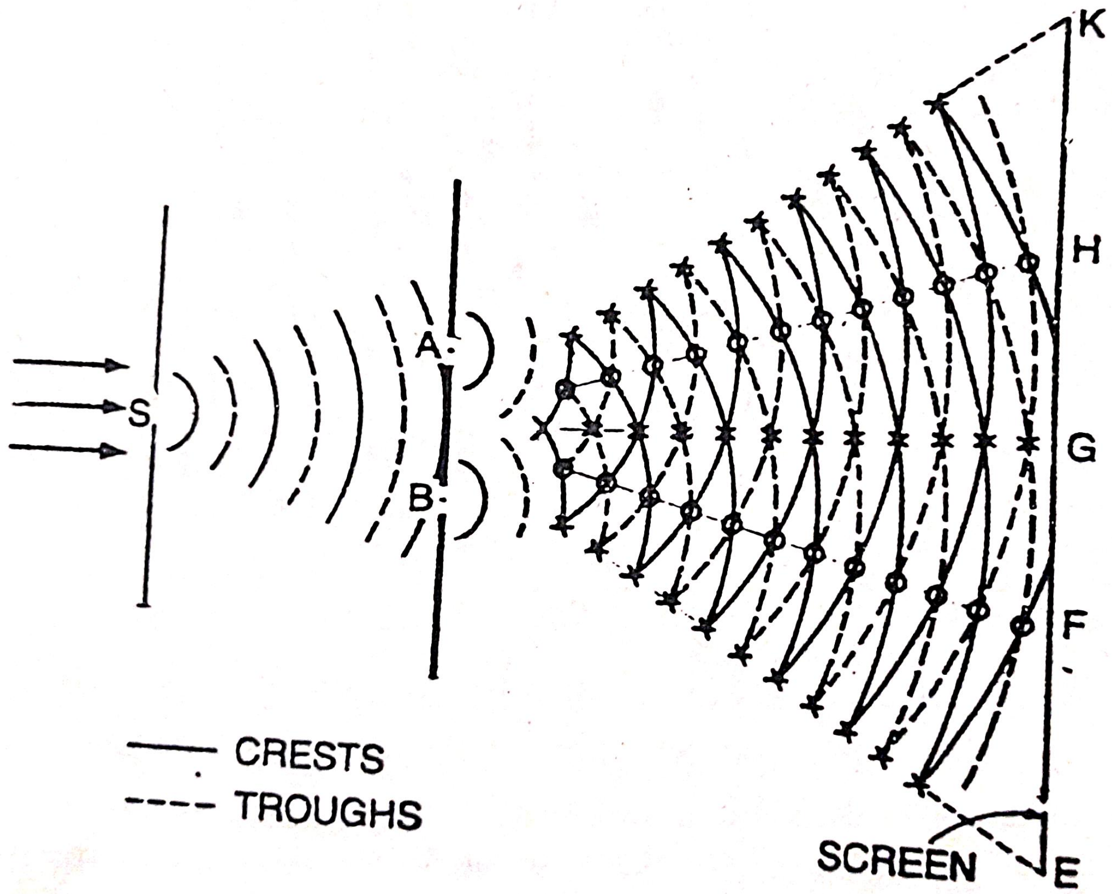


Fig. 8.2

A and *B* are equidistant from *S* and are close to each other. Spherical waves spread out from *S*. Spherical waves also spread out from *A* and *B*. These waves are of the same amplitude and wavelength. On the screen interference bands are produced which are alternatively dark and bright. The points such as *E* are bright because the crest due to one wave co-incides with the crest due to the other and therefore they reinforce with each other. The points such as *F* are dark because the crest of one falls on the trough of the other and they neutralize the effect of each other. Points, similar to *E*, where the trough of one falls on the trough of the other, are also bright because the two waves reinforce.

It is not possible to show interference due to two independent sources of light, because a large number of difficulties are involved. The two sources may emit light waves of largely different amplitude and wave-length and the phase difference between the two may change with time.

8.3 COHERENT SOURCES

Two sources are said to be coherent if they emit light waves of the same frequency, nearly the same amplitude and are always in phase with each other. It means that the two sources must emit radiations of the same colour (wavelength). In actual practice it is not possible to have two independent sources which are coherent. But for experimental purposes, two virtual sources formed from a single source can act as coherent sources. Methods have been devised where (*i*) interference of light takes place between the waves from the real source and a virtual source (*ii*) interference of light takes place between waves from two sources formed due to a single source. In all such cases, the two sources will act, as if they are perfectly similar in all respects.

Since the wavelength of light waves is extremely small (of the order of 10^{-5} cm), the two sources must be narrow and must also be close to each other. Maximum intensity is observed at a point where the phase difference between the two waves reaching the point is a whole number multiple of 2π or the path difference between the two waves is a whole number multiple of wavelength. For minimum intensity at a point, the phase difference between the two waves reaching the point should be an odd number multiple of π or the path difference between the two waves should be an odd number multiple of half wavelength.

8.4 PHASE DIFFERENCE AND PATH DIFFERENCE

If the path difference between the two waves is λ, the phase difference $= 2\pi$.

Suppose for a path difference x, the phase difference is δ

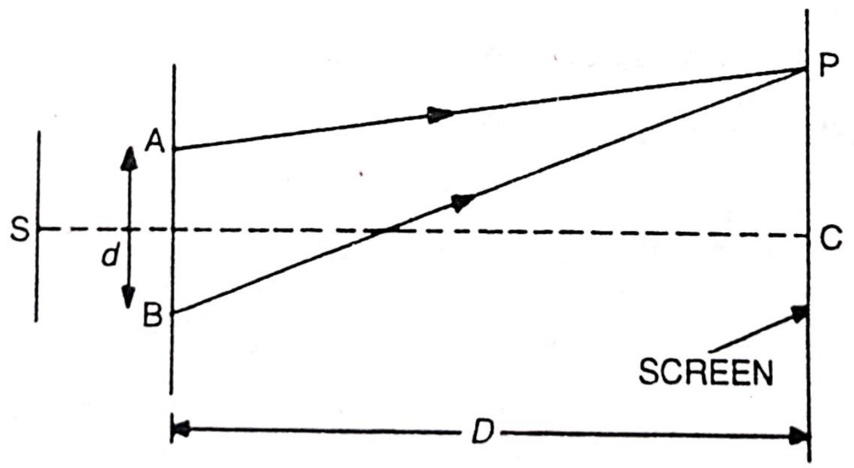
For a path difference λ, the phase difference $= 2\pi$

$\therefore$ For a path difference x, the phase difference $= \dfrac{2\pi x}{\lambda}$

Phase difference $\delta = \dfrac{2\pi x}{\lambda} = \dfrac{2\pi}{\lambda} \times (\text{path difference})$

8.5 ANALYTICAL TREATMENT OF INTERFERENCE

Consider a monochromatic source of light S emitting waves of wavelength λ and two narrow pinholes A and B (Fig. 8.3). A and B are equidistant from S and act as two virtual coherent sources. Let a be the amplitude of the waves. The phase difference between the two waves reaching the point P, at any instant, is δ.



If y_1 and y_2 are the displacements

$$y_1 = a \sin \omega t$$

$$y_2 = a \sin (\omega t + \delta)$$

$$\therefore \quad y = y_1 + y_2 = a \sin \omega t + a \sin (\omega t + \delta)$$

$$y = a \sin \omega t + a \sin \omega t \cos \delta + a \cos \omega t \sin \delta$$

$$= a \sin \omega t (1 + \cos \delta) + a \cos \omega t \sin \delta.$$

Taking $\quad a (1 + \cos \delta) = R \cos \theta \quad$ ...(*i*)

and $\quad a \sin \delta = R \sin \theta \quad$ ...(*ii*)

$$y = R \sin \omega t \cos \theta + R \cos \omega t \sin \theta$$

$$y = R \sin (\omega t + \theta) \quad \text{...(iii)}$$

which represents the equation of simple harmonic vibration of amplitude R.

Squaring (*i*) and (*ii*) and adding,

$$R^2 \sin^2 \theta + R^2 \cos^2 \theta = a^2 \sin^2 \delta + a^2 (1 + \cos \delta)^2$$

or
$$R^2 = a^2 \sin^2 \delta + a^2 (1 + \cos^2 \delta + 2 \cos \delta)$$
$$R^2 = a^2 \sin^2 \delta + a^2 + a^2 \cos^2 \delta + 2 a^2 \cos \delta$$
$$= 2a^2 + 2a^2 \cos \delta = 2a^2 (1 + \cos \delta)$$
$$R^2 = 2a^2 . 2 \cos^2 \frac{\delta}{2} = 4a^2 \cos^2 \frac{\delta}{2}$$

The intensity at a point is given by the square of the amplitude

$\therefore$
$$I = R^2$$

or
$$I = 4a^2 \cos^2 \frac{\delta}{2} \qquad ...(iv)$$

Special cases : (*i*) When the phase difference $\delta = 0, 2\pi, 2\,(2\pi), \ldots n\,(2\pi)$, or the path difference $x = 0, \lambda, 2\lambda, \ldots n\lambda$.
$$I = 4a^2$$

Intensity is maximum when the phase difference is a whole number multiple of 2π or the path difference is a whole number multiple of wavelength.

(*ii*) When the phase difference, $\delta = \pi, 3\pi, \ldots (2n + 1)\,\pi$, or the path difference $x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}, \ldots (2n + 1)\frac{\lambda}{2}$,
$$I = 0$$

Intensity is minimum when the path difference is an odd number multiple of half wavelength.

(*iii*) **Energy distribution**. From equation (*iv*), it is fount that the intensity at bright points is $4a^2$ and at dark points it is zero. According to

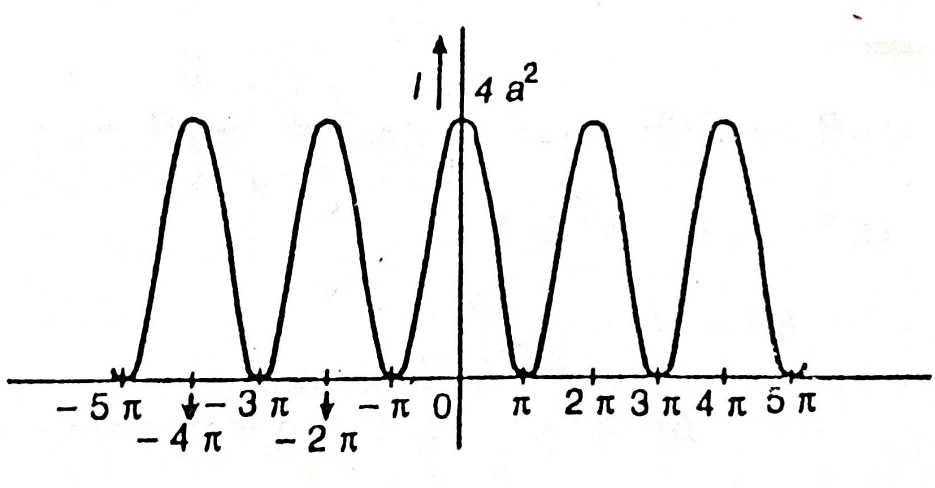


Fig. 8.4

the law of conservation of energy, the energy cannot be destroyed. Here also the energy is not destroyed but only transferred from the points of minimum intensity to the points of maximum intensity. For, at bright

The distance between any two consecutive dark fringes,

$$x_2 - x_1 = \frac{5\lambda D}{2d} - \frac{3\lambda D}{2d} = \frac{\lambda D}{d} \quad ...(vi)$$

The distance between any two consecutive bright or dark fringes is known as fringe width. Therefore, alternately bright and dark parallel fringes are formed. The fringes are formed on both sides of C. Moreover, from equations (*iv*) and (*vi*), it is clear that the width of the bright fringe is equal to the width of the dark fringe. All the fringes are equal in width and are independent of the order of the fringe. The breadth of a bright or a dark fringe is, however, equal to half the fringe width and is equal to $\frac{\lambda D}{2d}$. The fringe width $\beta = \frac{\lambda D}{d}$.

Therefore, (*i*) the width of the fringe is directly proportional to the wavelength of light, $\beta \propto \lambda$. (*ii*) The width of the fringe is directly proportional to the distance of the screen from the two sources, $\beta \propto D$. (*iii*) the width of the fringe is inversely proportional to the distance between the two sources, $\beta \propto \frac{1}{d}$. Thus, the width of the fringe increases (*a*) with increase in wavelength (*b*) with increase in the distance D and (*c*) by bringing the two sources A and B close to each other.